

```
golden_improved <- function(f, lower, upper, tol) {  
  ratio <- 2 / (3 + sqrt(5))  
  x1 <- (1 - ratio) * lower + ratio * upper  
  f.x1 <- f(x1)  
  while (abs(upper - lower) > 2 * tol) {  
    x2 <- (1 - ratio) * x1 + ratio * upper  
    f.x2 <- f(x2)  
    if (f.x1 < f.x2) {  
      upper <- lower  
      lower <- x2  
    } else {  
      lower <- x1  
      x1 <- x2  
      f.x1 <- f.x2  
    }  
  }  
  return((upper + lower) / 2)  
}
```